3/70

TRAFFIC ACCIDENT RESEARCH UNIT

DETERMINATION OF SWEPT PATHS OF VEHICLES

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The Traffic Accident Research Unit was established within the Department of Motor Transport, New South Wales, in May 1969 to provide a scientific approach into traffic accident prevention.

The Unit brings together a team of medical practitioners, scientists, statisticians, psychologists, sociologists and engineers engaged full time on research into all facets of road accident causation.

This paper is one of a number which report on their research and is published for the information of all those interested in the prevention of traffic accidents.

D. R. Coleman

Commissioner.

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**3/70
JULY, 1970**

ABSTRACT

Determination of Swept Paths of Vehicles

When a vehicle, after travelling in a straight line, is driven along a curve, the path swept by the vehicle increases in width. This paper describes a means by which the swept paths of vehicles may be determined by means of a mathematical model; the method also enables the path followed by any point on the vehicle to be described in mathematical or graphical form.

Discussion is given of various aspects of the practical application of the method. Experiments have indicated that the method describes vehicular swept paths with an error of less than half of one percent.

Some graphical examples of swept paths of vehicles are included.

INTRODUCTION

When a vehicle, after travelling in a straight line, is driven along a curve, the path swept by the vehicle increases in width. If a curve of constant radius is followed, the vehicle's swept path approaches a constant width.

Swept paths provide a means for comparison of new or proposed vehicles with those that are already being driven along existing road curves. Swept paths may also be used to provide data for the design of road layouts. They may be determined by measuring the paths described by actual vehicles or by models of vehicles. The first of these methods is not possible when a vehicle is in the design stage only, and both are very time consuming.

This paper describes a means by which the swept paths of vehicles may be determined using a mathematical model. It is concerned solely with the physical attributes of the vehicle and takes no account of any difficulties that may be encountered by drivers in curve following.

EVOLUTION OF THE THEORY

Basic Assumptions

The basis of the theory is the assumption that, at any instant in the progression of a point on a motor vehicle along a curved path, the instantaneous centre of the curve always lies on a line projected through the axis of the centre of the non-steering axle or group of axles on the vehicle, as shown in Figure 1.

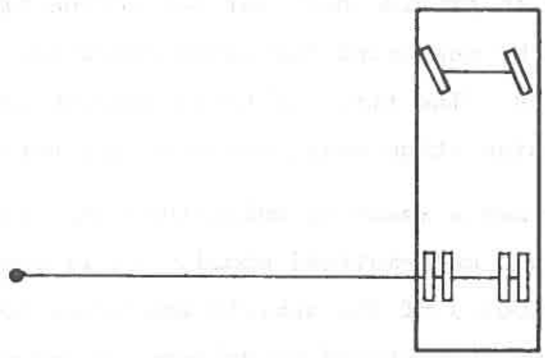


Figure 1

For vehicles other than rigid vehicles, the instantaneous centre about which an individual section of the complete vehicle turns lies in general on a line projected through the axis of the centre of the non-steering axle on that vehicle section. An illustration of the positions of instantaneous centres is given in Figure 2. Points X, Y and Z are the instantaneous centres for the truck, dolly and trailer respectively; these points would coincide if the vehicle were in an equilibrium position.

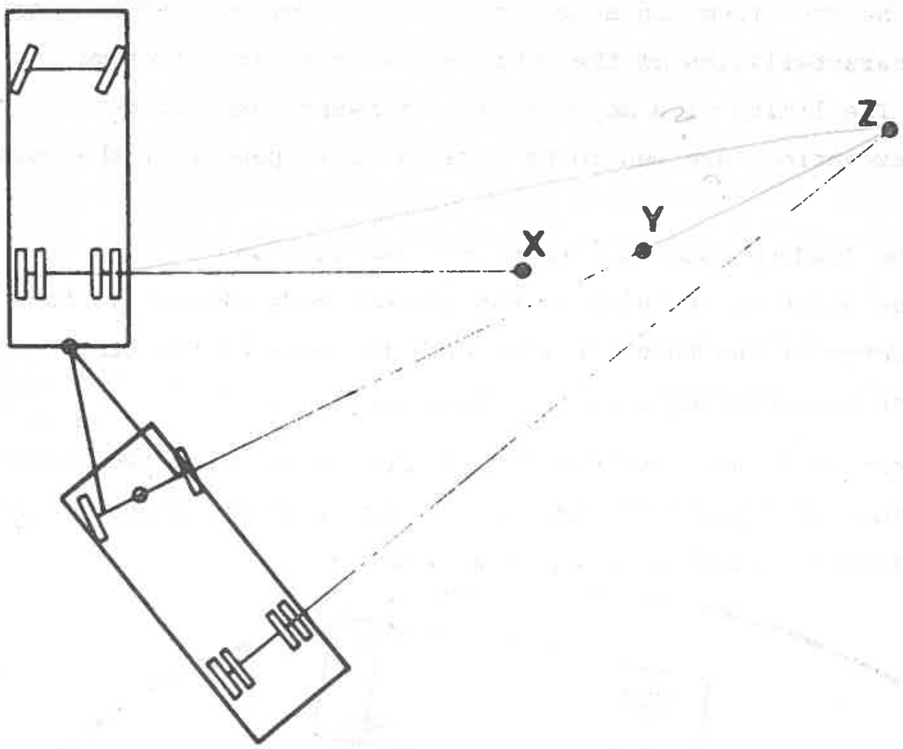


Figure 2

The further assumption has been made that vehicles do not experience any significant skidding or weight transfer on cornering.

<u>Data Required</u>

The path swept by any motor vehicle on a curve is determined by:

(a) the angle between the approach and departure paths (usually tangential) of the curve;

(b) the radius of the path taken by the leading outer corner of the vehicle;

(c) the type of vehicle negotiating the curve (rigid, articulated, rigid and trailer combination, articulated and trailer combination, road-train);

(d) the dimensions of the vehicle.

It may be seen from the above that, for any given road situation, the design characteristics of the vehicle concerned are of prime importance. The limits of a motor vehicle's swept path are generally determined (exceptions are mentioned later in this paper) by the loci of:

(i) the leading outer corner of the vehicle; and

(ii) the point on the side of the vehicle body closest to the centre of the turn, in line with the axis of the centre of the non-steering rear axle or group of axles.

The swept path thus consists of the area between the two loci. Examples of the swept paths of various vehicle types are diagrammatically represented (in plan view) in Figures 3, 4 and 5.

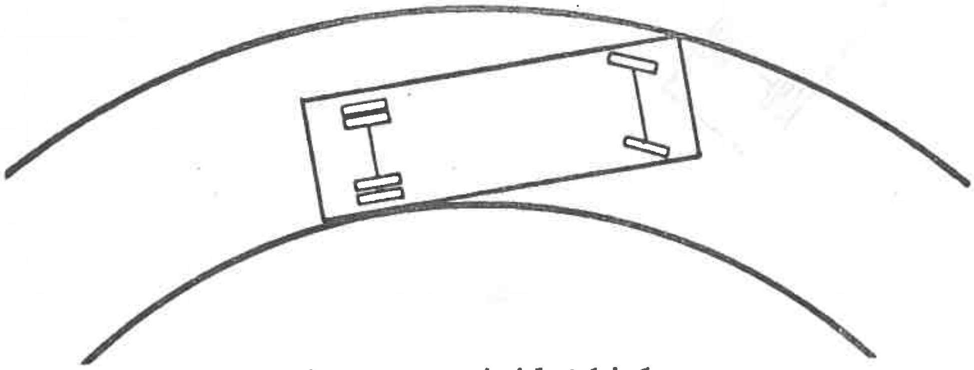


Figure 3 - Rigid Vehicle

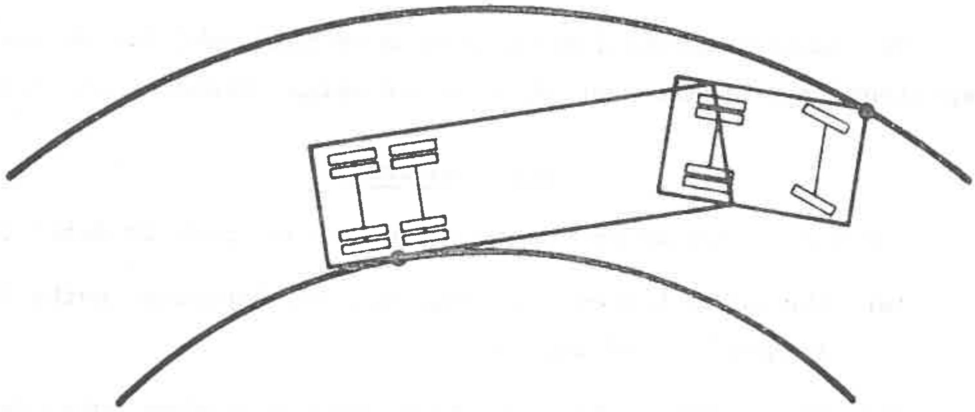


Figure 4 - Prime Mover and Semi-trailer

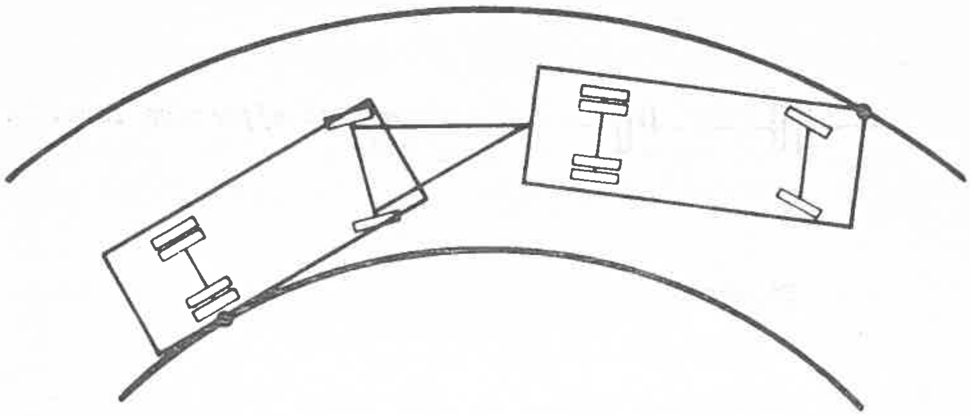


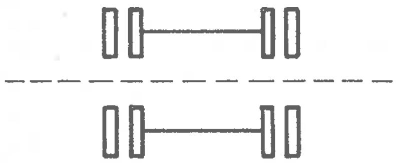
Figure 5 - Rigid Vehicle and Dog Trailer

The theory expounded in this paper for determining vehicle swept paths is dependent upon a specification of the path, defined as the Reference Curve, travelled by a point, defined as the Reference Point on the vehicle. Although the Reference Point may be anywhere on the vehicle, in practice, it is usually located at either the leading outer corner of the vehicle or the position of the outer edge of the outer front tyre at its axis.

One fundamental requirement for application of this theory is knowledge of the location of the non-steering axle, or its effective location where there are two or more non-steering axles. It has been assumed that the effective location of the vehicle's non-steering axle in the latter case is the geometric centre of the tyre and axle group as illustrated in Figures 7 and 8.

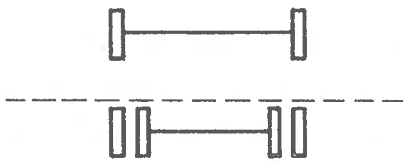
actual and effective location of axle

Figure 6



effective location of axles

Figure 7



effective location of axles

***Figure** 8*

DETERMINATION OF THE SWEPT PATH OF A RIGID VEHICLE

Consider a rigid vehicle turning through a curve, as represented in plan view in Figure 9. The Reference Point A is a selected fixed point on the vehicle, and the line AB is a line through A perpendicular to the effective non-steering axle. Now any point on the vehicle may be defined in terms of a distance from the Reference Point A and an angle to the line AB. For example the point C in Figure 10 is defined by the angle Ø and the dimension AC. The locus of point C in this case represents the inner limit of the vehicle's swept path.

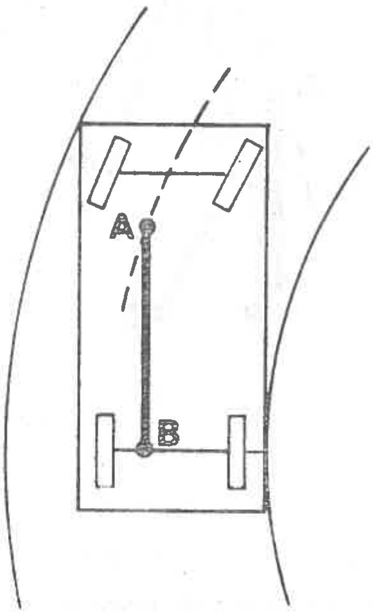


Figure 9

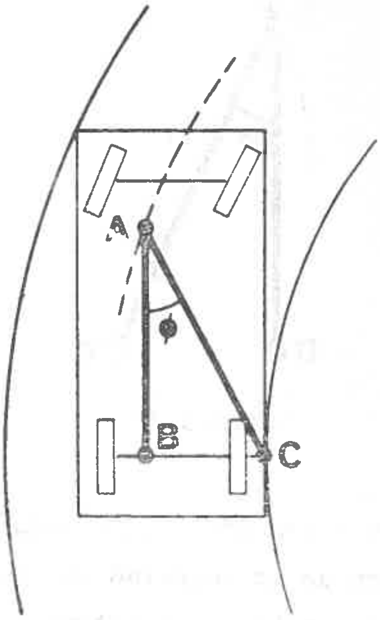


Figure 10

Now consider the problem of determining the actual inner limit of the swept path. The required vehicle data are represented in diagrammatic form in Figure 11. If the Reference Point A is moved a small increment along the Reference Curve to a new position A^1, then the points B and C will move to new positions B^1 and C^1 respectively. It has been assumed that, for a small incremental movement, the point B^1 will lie on the line A^1B, thus determining the new position of C that is C^1. This is shown in Figure 12. The movement from C to C^1 now forms an increment of the required inner limit to the swept path.

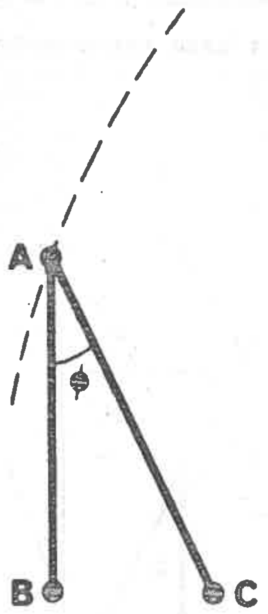


Figure 11

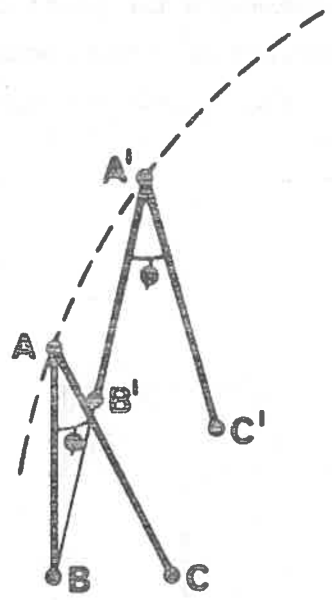


Figure 12

This incremental procedure is repeated as shown in Figure 13 as often as is required in order to generate the desired locus. Any curve consists of an infinite number of chords and the theory in this paper depends for its accuracy upon the choice of increment. By utilising increments of a suitable size, highly accurate results may be achieved as demonstrated in Figure 20 of the Appendix to this paper.

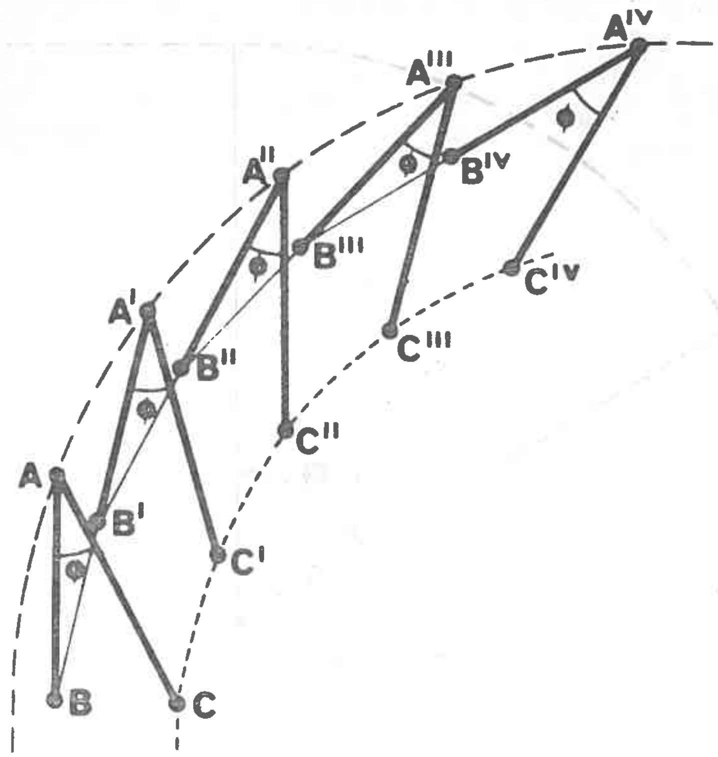


Figure 13

It will be recalled that the path travelled by the Reference Point A is known. For most cornering situations, point A travels in a straight line into the curve, along an arc of known radius and angle in the apex of the curve, and a straight line out of the curve. This type of path is reproduced in Figure 14. Thus from a knowledge of the vehicle's initial position (it is usually assumed that the vehicle enters the curve parallel to the initial straight section of the Reference Curve as represented in Figure 14), and by numerous simple geometrical calculations, the incremental movement of point C on the vehicle may be calculated and thus the inner limit to the swept path may be obtained.

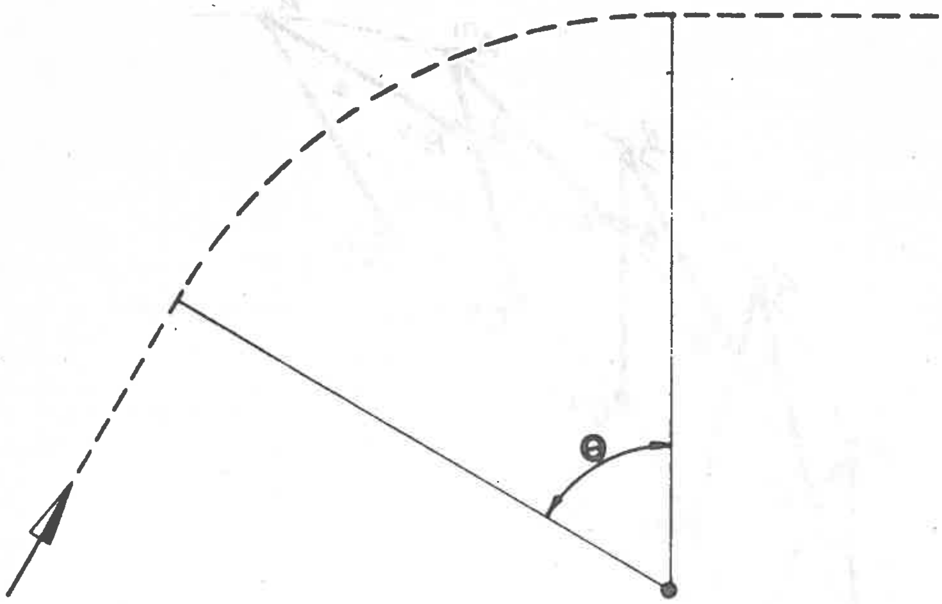


Figure 14

In a similar manner to the above, any required point on the vehicle may be defined in relation to the Reference Point, and its locus obtained. Thus the locus of the leading outer corner (point D in Figure 15) of the vehicle may be determined, defining the outer limit of the vehicle's swept path, as illustrated in Figure 15.

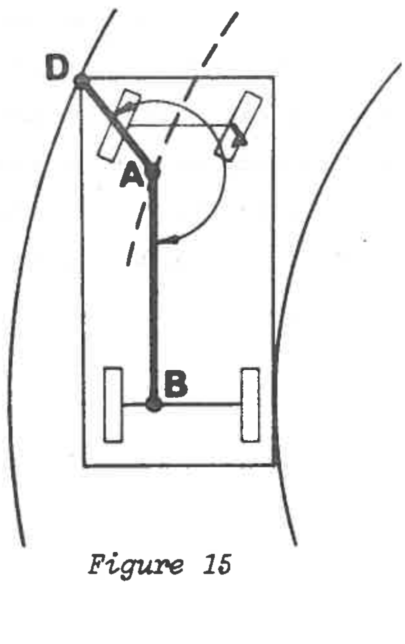


Figure 15

DETERMINATION OF THE SWEPT PATH OF AN ARTICULATED VEHICLE

Consider an articulated vehicle having a single pivot point as illustrated in Figure 4. Now the prime mover can be considered simply as a rigid vehicle. The Reference Point for the complete vehicle is, in practice, always located upon the prime mover.

It has been demonstrated in the previous section that, for a given Reference Curve, the locus of any point on a rigid vehicle may be obtained. Thus the locus of the pivot point on the prime mover can be obtained. Now the trailer section of the vehicle may also be considered as a separate rigid vehicle, its Reference Point being the pivot common to prime mover and trailer, the locus of which is known. This enables the determination of the locus of any point on the trailer. If the point F on Figure 16 is plotted, it then constitutes the inner limit of the swept path of the complete articulated vehicle.

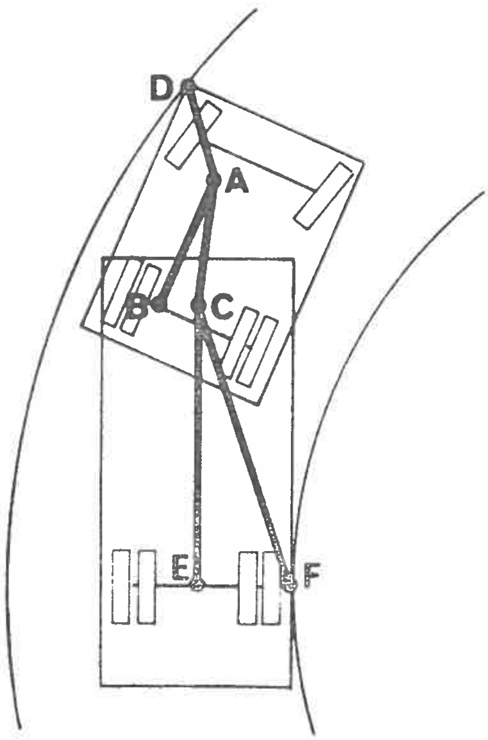


Figure 16

The outer limit of the swept path is usually defined by the leading outer corner of the prime mover (point D in Figure 16), and this is obtained in the manner described for rigid vehicles. In cases where the prime mover is narrow and the trailer wide, the leading outer corner of the trailer may dictate the outer limit; this can be checked by determining both this point and the leading outer corner of the prime mover.

DETERMINATION OF THE SWEPT PATHS OF MULTI-JOINTED VEHICLES

A multi-jointed vehicle may consist of a truck and trailer combination with two pivot points (as in Figure 5), an articulated vehicle and trailer combination ("doubles") with three pivot points (as in Figure 17), or a road train with a larger number of pivot points. For the purposes of this theory the Reference Point for such vehicles must be chosen so as to lie on the prime mover.

Now it has been demonstrated that the locus of the first pivot point of the combination (that is, on the prime mover) can be obtained. This enables the locus of any point on the vehicle section coupled to that first pivot point (that is, the first trailer section following the prime mover) to be found, for example the locus of the second pivot point can also be found. This in turn enables any point on the vehicle section coupled to the second pivot point to be found and this may be yet another pivot point.

This procedure may be continued until the last section of the vehicle is reached, whereupon the locus of the point adjacent to the body (on the inner part of the curve) in line with the rear axle may be plotted and thus the inner limit of the swept path obtained. The outer limit of the swept path of the vehicle combination is usually the outer leading corner of the prime mover, as previously noted.

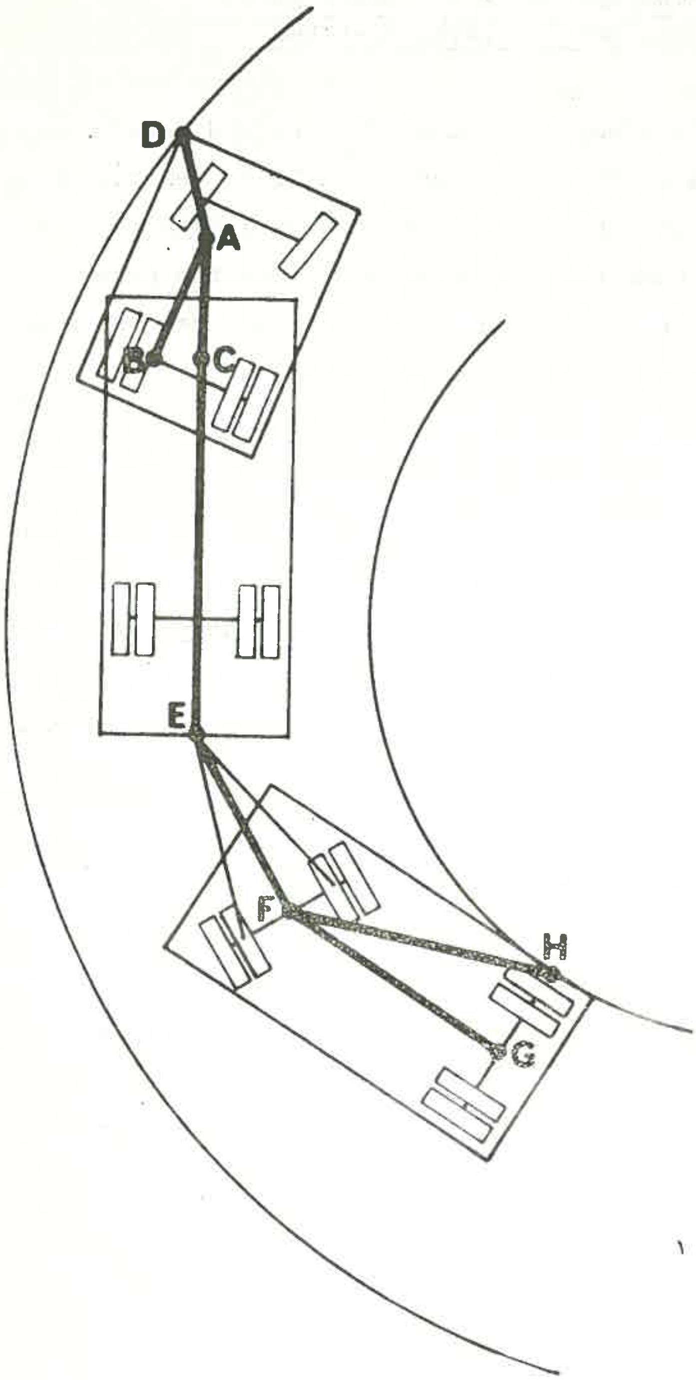


Figure 17

APPLICATION OF THE METHOD

Choice of Reference Point and Reference Curve

As will be apparent from the preceding sections, it is a prerequisite for application of the theory for plotting swept paths that the Reference Point and Reference Curve be defined for any vehicle and road situation under consideration. Now swept paths are mainly of interest where there is a possibility that the path will exceed a permissible limit.

In most cases, the Reference Point on the vehicle under consideration is the leading outer corner, since its locus usually constitutes the outer limit of the vehicle's swept path. This point is easily located and referred to by the driver during any manoeuvre of the vehicle. When the Reference Point is located in this way the Reference Curve is usually an arc of the vehicle's minimum turning circle (as described by the vehicle body) or the outer limit of the curve in the road being considered, whichever has the greater radius (see Figure 18). It should be noted that the Reference Curve need not be a curve with a constant radius: multi-radii Reference Curves can be readily utilised provided that all of the data needed to describe the curve is available.

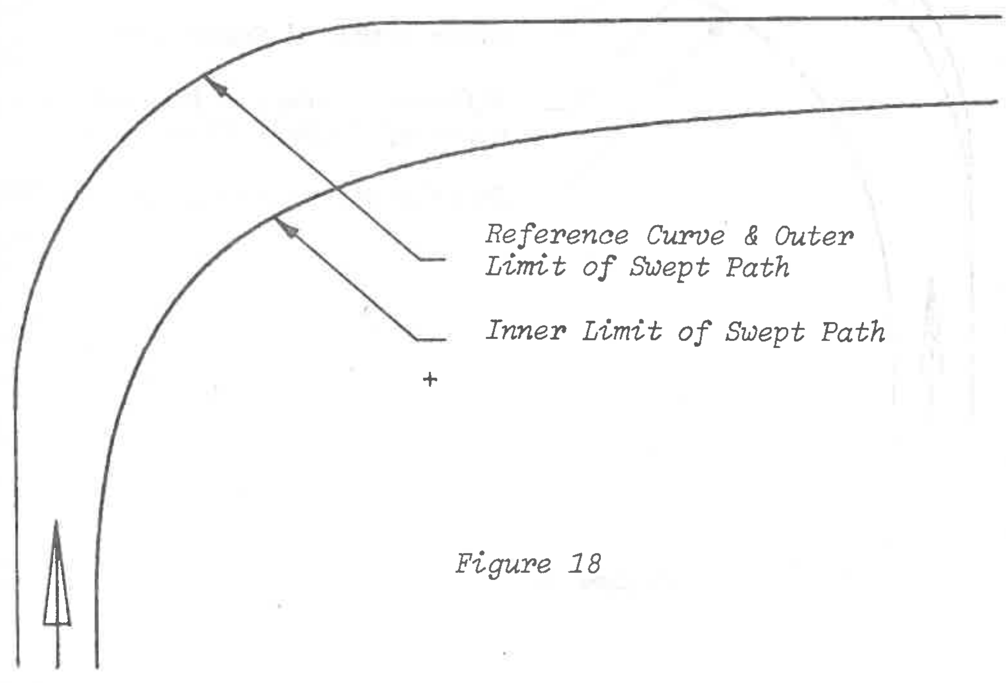


Figure 18

There may be cases where vehicles' swept paths may be permitted to deviate slightly from the area defined by the actual road surface, provided that the vehicles' tyres remain on the road surface. As an example of usch a case, an omnibus with a large body overhang forward of the front axle may be considered. The body of such a vehicle could be permitted to overhang the unsealed verges of a curve in a road, provided that the tyres of the vehicle remained on the sealed road surface. In such a case, the Reference Point would be the outer edge of the outer front tyre at its axis, and the Reference Curve would be the outer edge of the sealed road surface, or an arc of the minimum turning circle of the vehicle (measured at the tyres), whichever is the greater (see Figure 19). It should be noted that legislation might limit the maximum turning circles of vehicles, defined by either the area described by either the tyres (as is the case of New South Wales legislation) or by the vehicle's body.

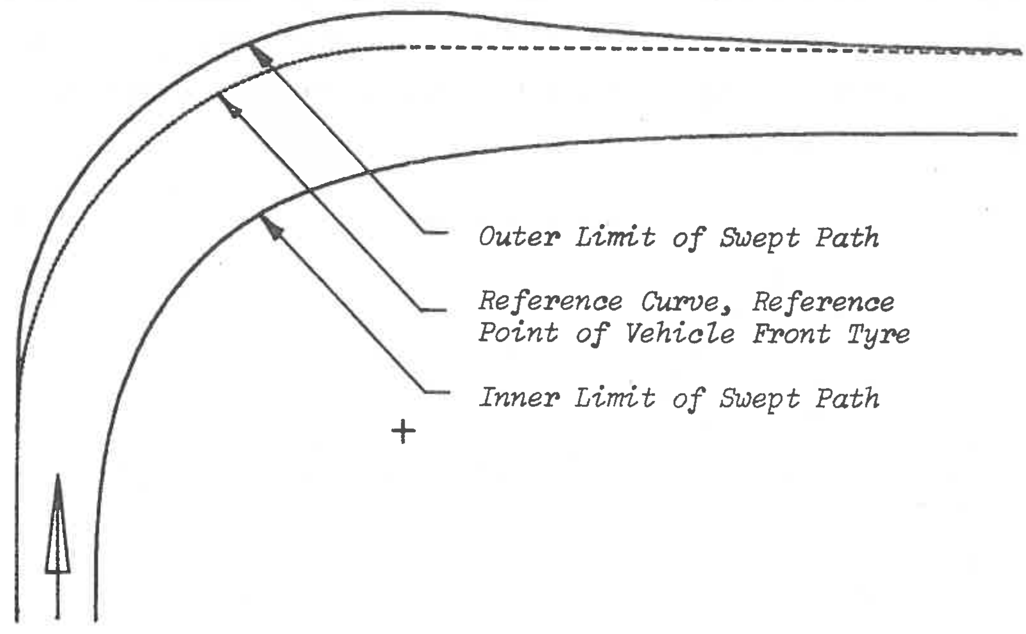


Figure 19

It is the opinion of the authors that it is preferable and more realistic to define the Reference Point on any vehicle as the point which describes the outer limit of the vehicle's swept path, and the Reference Curve as the outer limit of the curve concerned (if this is within the turning capabilities of the vehicle). This means that, in the majority of cases, the outer leading corner of the vehicle or prime mover becomes the Reference Point and either the outer limit of the curve or an arc of the minimum turning circle of the vehicle, whichever is the greater, becomes the Reference Curve.

Swept Path Effects of Vehicle Rear Outer Corners

In most examples of vehicle swept paths, the rear outer corner of the vehicle describes the outer limit of the swept path in the initial stages only. This effect is usually quite small and in most cases can be comfortably ignored. It is only when unusual vehicles or critical initial path limitations are considered that the path swept by the rear outer corner of the vehicle is of importance. An illustration of this effect is given in Figures 24 and 25 of the Appendix.

Unconventional Steering Arrangements

Throughout the preceding pages, references to vehicles have been restricted to those vehicles with conventional axle layouts, that is, to vehicles having steering front axles and non-steering rear axles. The theory can be equally well applied to more unusual axle layouts (with, for example, the conventional axle layouts reversed, as in the case of some fork lift trucks), provided the line upon which the instantaneous centre of the Reference Curve always lies (and which line is in a constant position relative to the vehicle) can be determined. Examples of the swept paths of unusual vehicles are included in the Appendix.

Form of Swept Path Output

By using a programmable calculator and suitable programming techniques, the theory enables swept paths of vehicles to be described in graphical or mathematical form. Mathematical outputs may be produced at the conclusion of each incremental movement along the locus concerned, in either rectangular or polar co-ordinate forms.

General Comments

This paper describes a tool which may be used in various phases of traffic and transportation planning, and which is particularly useful in the motor vehicle/road context. It can, for example, be used to show that some large vehicles currently permitted to be used in central business districts cannot turn at some intersections whilst remaining in one (kerbside) traffic lane. More importantly, the method described provides a simple and economical means of determining the necessary vehicle and roadway design changes required to provide an optimised solution to this and to similar problems.

Accuracy of the Method

In order to check the accuracy of the method, swept paths actually described by a number of vehicles have been compared with swept paths calculated for those vehicles. In the comparison, the results agreed to within half of one percent. A second, more theoretical check of accuracy is given in Figures 20 and 21 of the Appendix.

Vehicle Dimensions

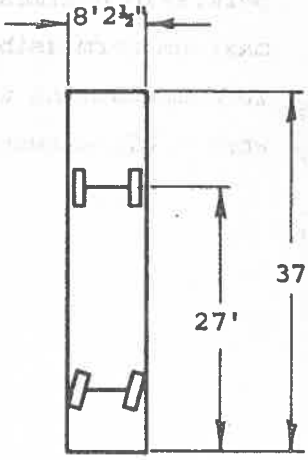
A research project currently in the planning stages in the Traffic Accident Research Unit will examine many aspects of motor vehicle dimensions, using the method of describing vehicle swept paths as the principal tool.

One object of the project is to produce recommendations for maximum permissible dimensions for all types of motor vehicles, based on a common maximum permissible swept path width. In addition, it is envisaged that recommendations will be made to enable the optimisation of roadway designs when vehicle swept paths are critical.

APPENDIX

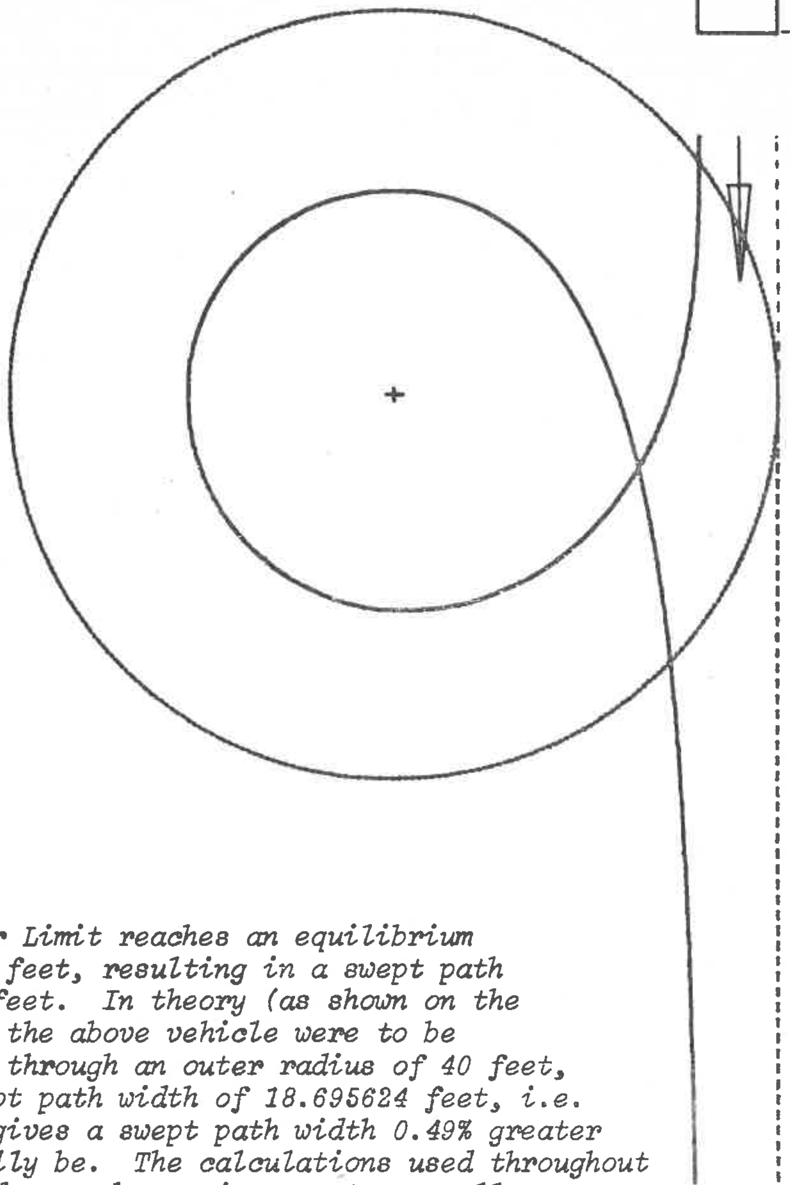
EXAMPLES OF SWEPT PATHS

Figure 20



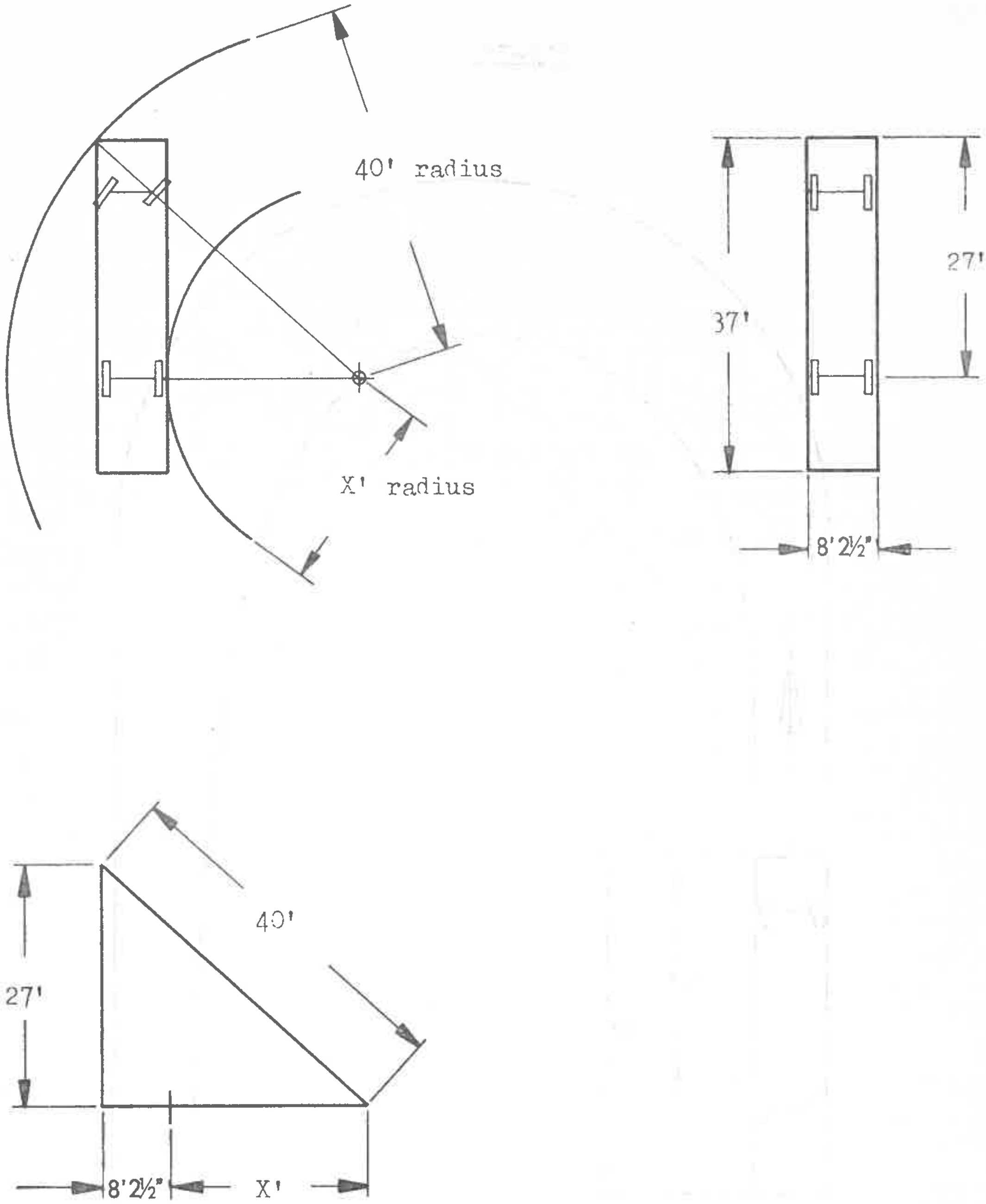
Test of Accuracy of Method

Scale	*1 inch*	=	*20 feet*
Outer Radius (Reference Curve)		=	*40 feet*
Angle of Curve		=	*360 degrees*



The Swept Path Inner Limit reaches an equilibrium radius of 21.211969 feet, resulting in a swept path width of 18.788031 feet. In theory (as shown on the following page), if the above vehicle were to be continuously turned through an outer radius of 40 feet, it would have a swept path width of 18.695624 feet, i.e. the subject method gives a swept path width 0.49% greater than it would actually be. The calculations used throughout this paper are based on ½ degree increments: smaller increments would result in greater accuracy.

Figure 21



Theoretical Swept Path of a Vehicle having a Constant Turning Radius

Scale   *1 inch = 20 feet*

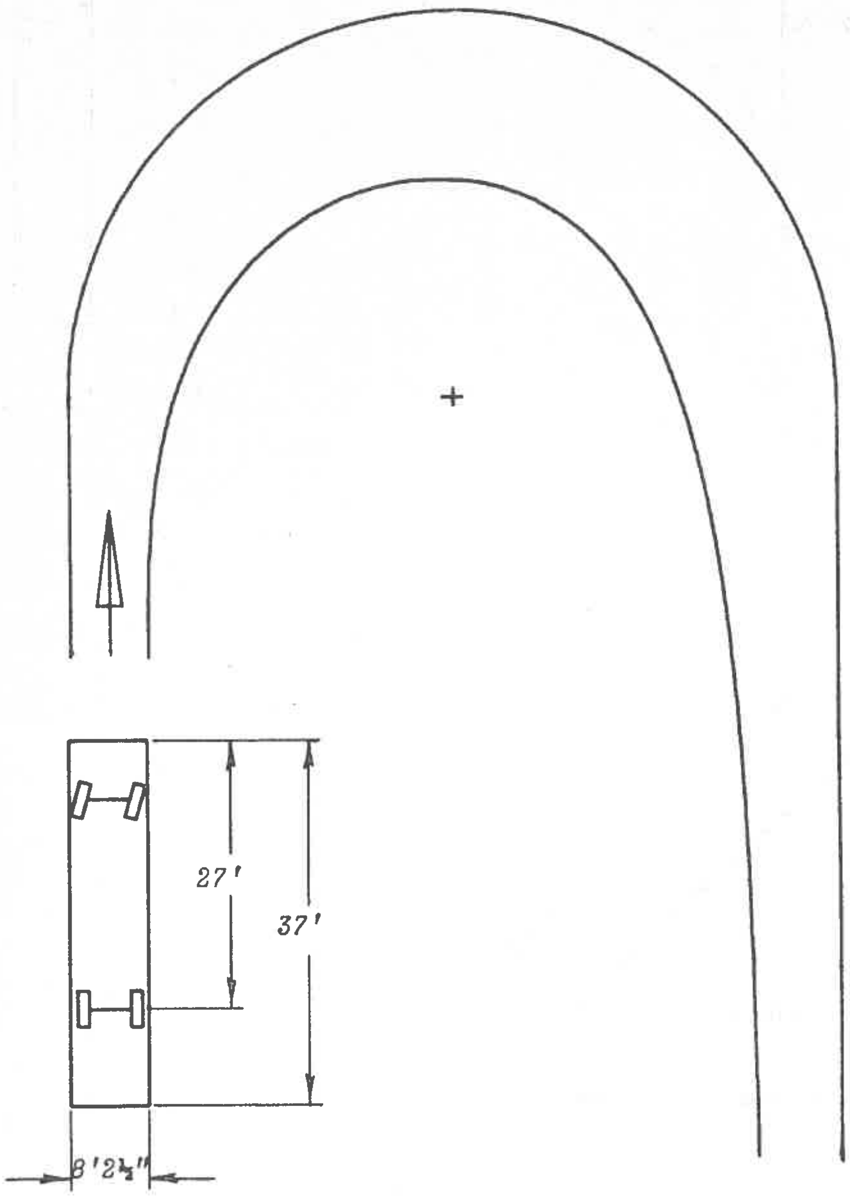
The swept path width is given by (40 - X) feet,

where $X = \sqrt{(40^2 - 27^2)} - 8.208333$

$= 21.304376$

i.e. swept path width $= 18.695624$ *feet*

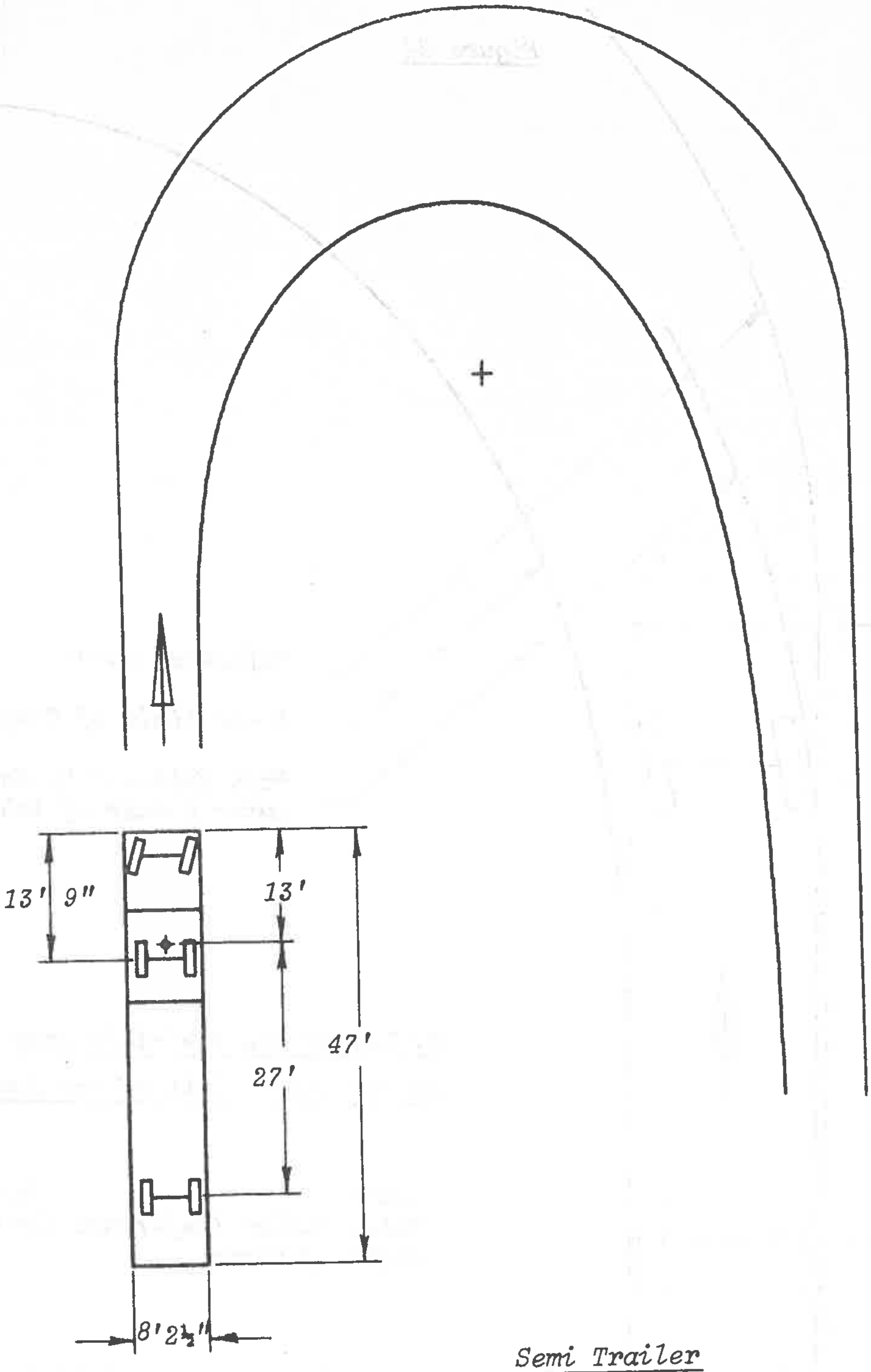
Figure 22



Rigid Vehicle

Scale	1 inch	= 20 feet
Outer Radius (Reference Curve)		= 40 feet
Angle of Curve		= 180 degrees
Swept Path Maximum Width		= 18.34 feet

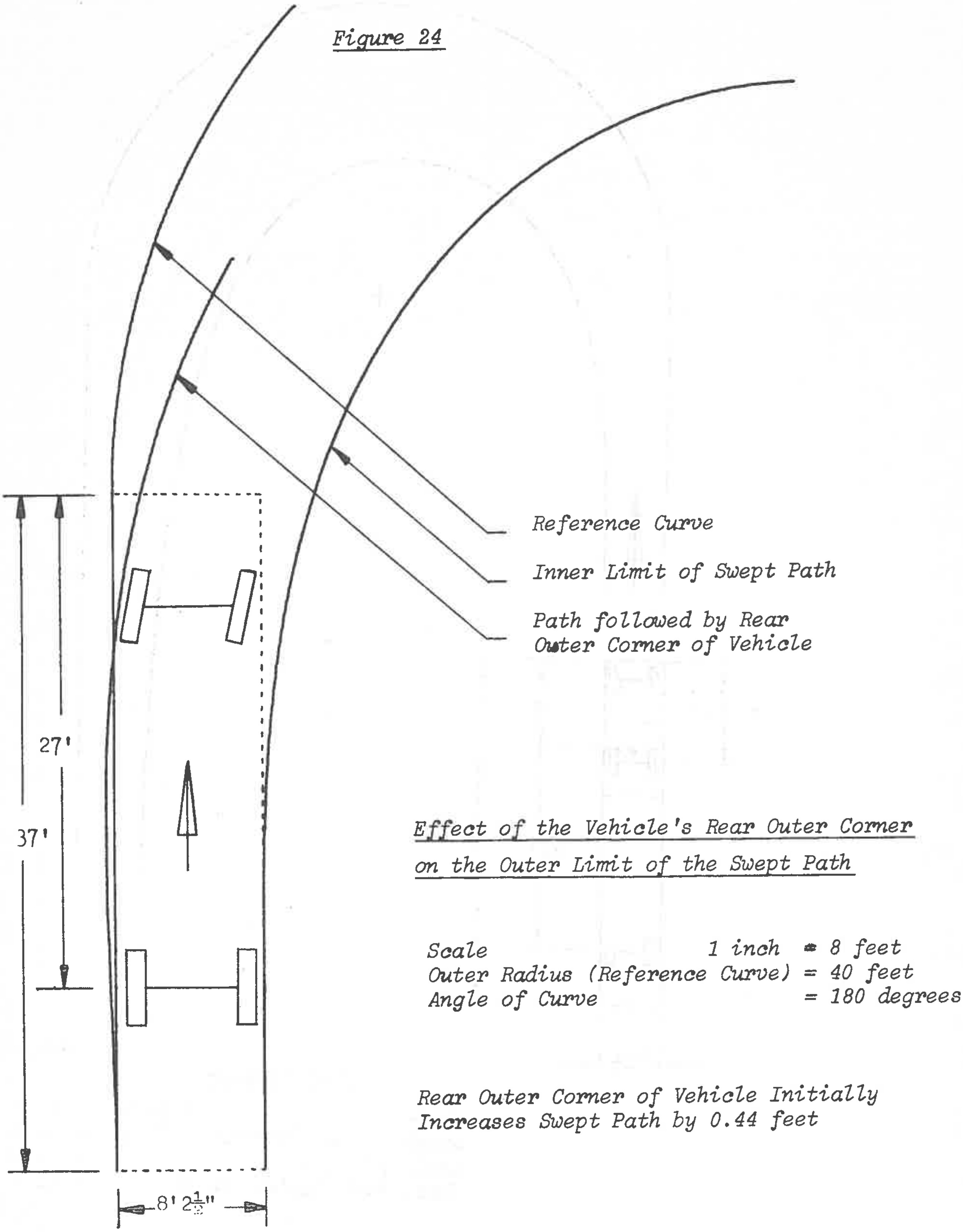
Figure 23



Semi Trailer

Scale 1 inch = 20 feet
Outer Radius (Reference Curve) = 40 feet
Angle of Curve = 180 degrees
Swept Path Maximum Width = 22.64 feet

Figure 24

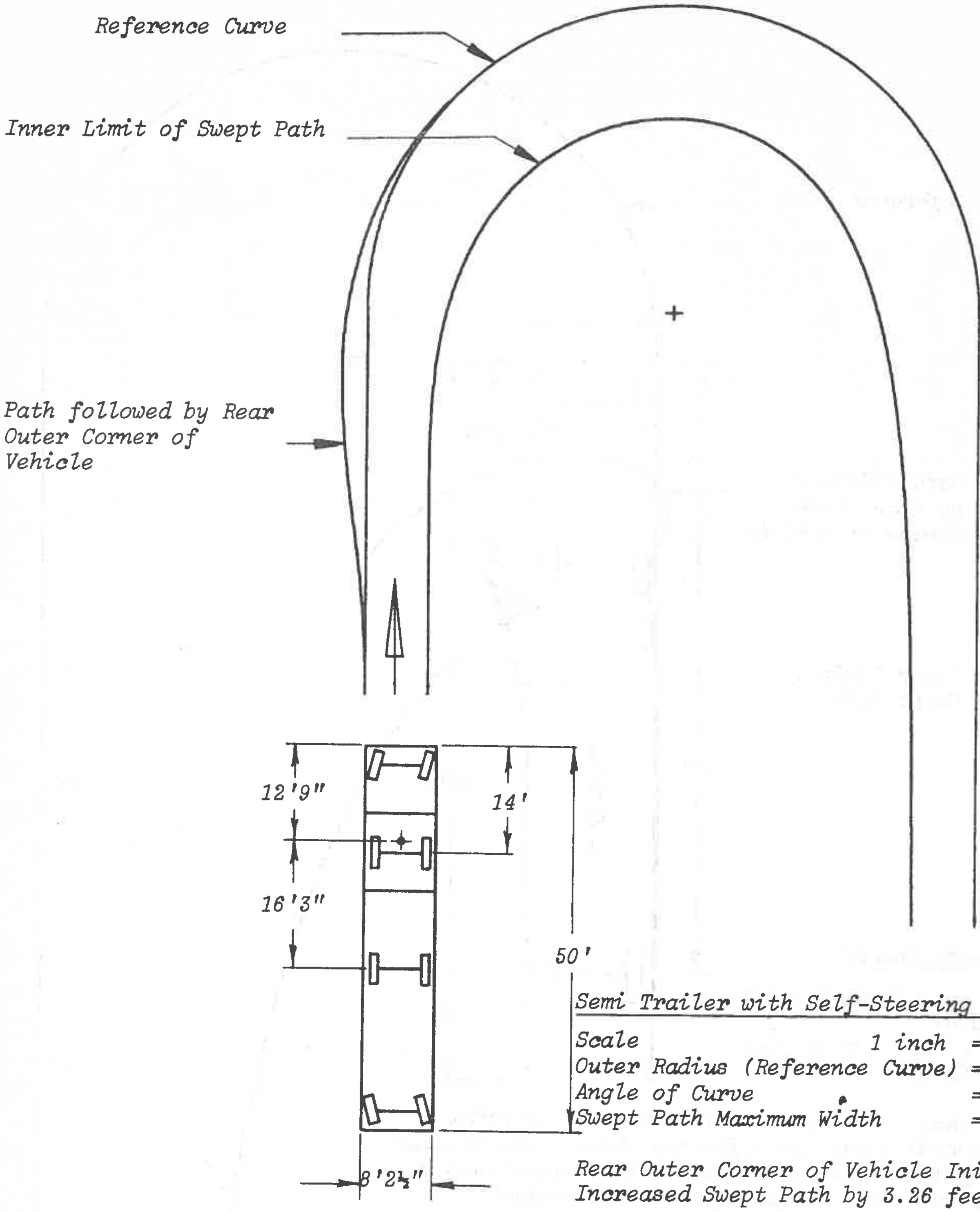


Effect of the Vehicle's Rear Outer Corner on the Outer Limit of the Swept Path

Scale 1 inch = 8 feet
Outer Radius (Reference Curve) = 40 feet
Angle of Curve = 180 degrees

Rear Outer Corner of Vehicle Initially Increases Swept Path by 0.44 feet

Figure 25

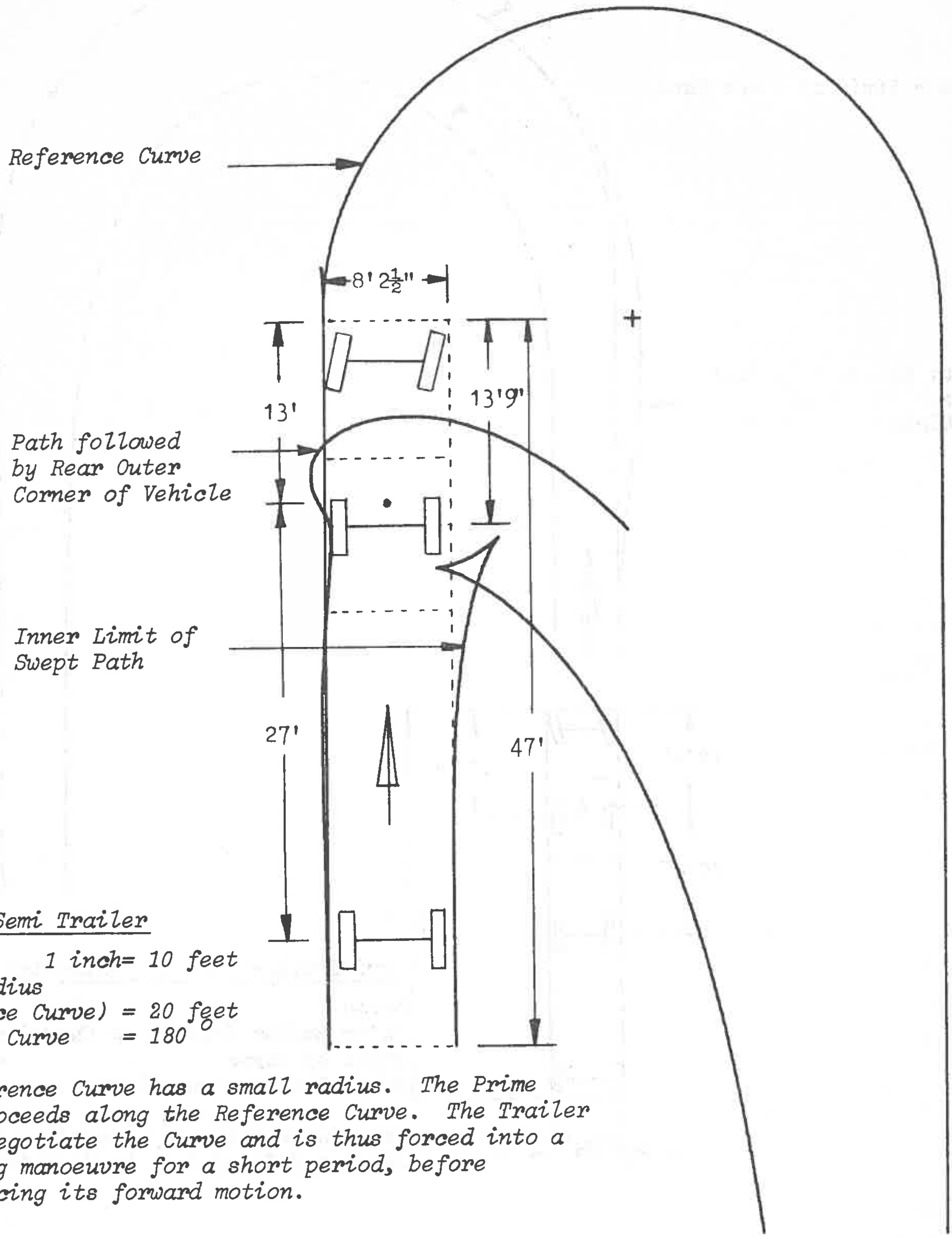


Semi Trailer with Self-Steering Trailer

Scale 1 inch	= 20 feet
Outer Radius (Reference Curve)	= 40 feet
Angle of Curve	= 180°
Swept Path Maximum Width	= 14.99 ft

Rear Outer Corner of Vehicle Initially Increased Swept Path by 3.26 feet.

Figure 26

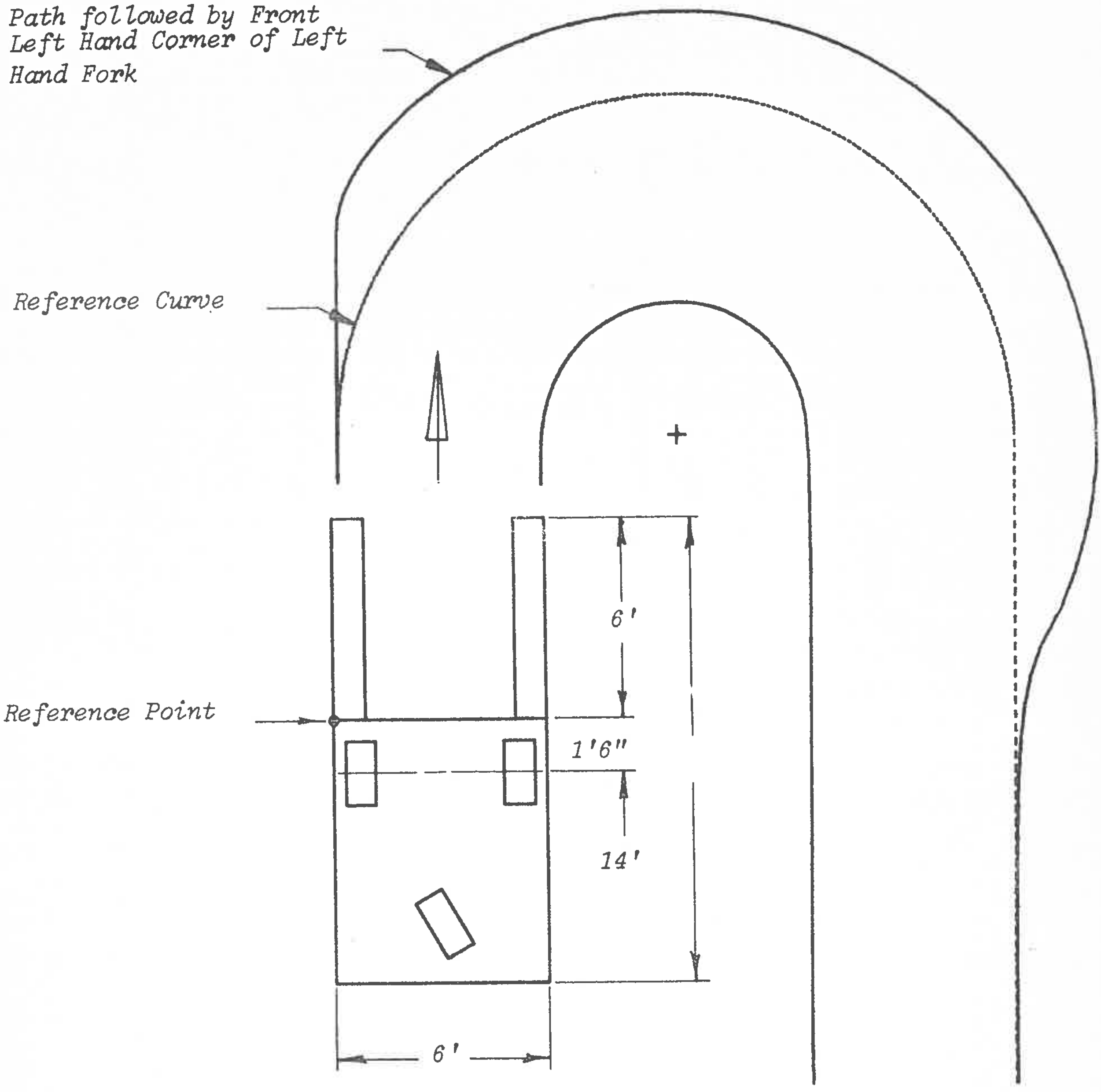


Semi Trailer

Scale 1 inch= 10 feet
Outer Radius
(Reference Curve) = 20 feet
Angle of Curve = 180°

The Reference Curve has a small radius. The Prime Mover proceeds along the Reference Curve. The Trailer cannot negotiate the Curve and is thus forced into a reversing manoeuvre for a short period, before recommencing its forward motion.

Figure 27



Fork Lift

Scale 1 inch = 5 feet
Outer Radius (Reference Curve) = 10 feet
Angle of Curve = 180 degrees
Swept Path Maximum Width = 8.54 feet

Radius followed by corner of Fork is 2.43 feet greater than Reference Radius.